

# Charmed Scalar Meson Production in $B$ Decays

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The study on the charmed scalar meson spectroscopy has become a hot topic both experimentally and theoretically. The  $B_{(s)}$  decays provide an ideal place to study their property. We employ the  $B$ -meson light-cone sum rules to compute the  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)$  and  $B^- \rightarrow D_0^{*0}(2400)$  transition form factors at large recoil, assuming  $D_s^{*+}(2317)$  and  $D_0^{*0}(2400)$  being scalar quark-anti-quark states. The results are extrapolated to the whole momentum region with the help of HQET. Considering large uncertainties, our results can be consistent with the previous studies, while the power corrections should be large. We also estimate the semi-leptonic decays  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)l\bar{\nu}_l$  and  $B^- \rightarrow D_0^{*0}(2400)l\bar{\nu}_l$ . The branching fraction of the semi-leptonic  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)l\bar{\nu}_l$  decay is around  $6 \times 10^{-3}$  for light leptons and  $0.8 \times 10^{-3}$  for tau final state. The predicted branching ration of  $B^- \rightarrow D_0^{*0}(2400)l\bar{\nu}_l$  is slightly larger than  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)l\bar{\nu}_l$ , and we hope the future data in LHCb can test these results.

PACS numbers: 14.40.Lb, 13.20.He, 11.55.Hx

## I. INTRODUCTION

The charmed scalar meson spectroscopy has evoked many interests since the observation of  $D_{s0}^*(2317)$  by Babar collaboration at 2003. In addition, the signal for the isospin doublet  $D_0^*(2400)$  has also been reported by Belle[2] and Focus[3] in the  $D\pi$  final state. Recently more measurements on the charmed scalar meson final state in  $B$  decays have been performed[4]. The low mass and the narrow width of  $D_{s0}^*(2317)$  indicates some hints on its mysterious inner structure. It is regarded as a scalar meson state in some studies, while it has also been assigned to be a four-quark state or the molecular state. Until now, the structure of  $D_{s0}^*(2317)$  is still a controversial problem. As for  $D_0^*(2400)$ , there is less information from experiments, and our knowledge of its property is even poorer. So we need more phenomenological analysis to clarify the inner structure of these  $p$ -wave states.

A great number of  $B$  decay events have been accumulated at  $B$  factories which provide good places to test the inner structure of the charmed scalar meson. To study the  $B$ -to-scalar meson decay modes theoretically, an essential task is to evaluate  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)$  and  $B^- \rightarrow D_0^{*0}(2400)$  transition form factors. In heavy quark effective theory(HQET)[5], the heavy-to-heavy form factor can be reduced to the universal Isgur-Wise(IW) function  $\xi(v \cdot v')$  in the heavy quark limit. In order to estimate the form factors or the IW function, one must employ the non-perturbative methods. There have existed some phenomenological studies using different approaches, including the phenomenological model [6], the QCD sum rules approach [7–9], PQCD approach [10], Lattice QCD [11–13], as well as the light-cone sum rules (LCSR)[14].

LCSR [15–17] combines the traditional QCD sum rules [18] with the theory of hard exclusive process, and offers a systematic way to compute the soft contribution to the transition form factor. The vacuum-to-hadron correlation function is computed in terms of light-cone OPE in the LCSR. In the conventional LCSR for  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)$  form factor, the correlation function is taken between the vacuum and  $D_s^{*+}(2317)$  state, whereas the  $B$  meson is interpolated by a local current. The long distance effect of the form factor is then described by the distribution amplitudes(DAs) of  $D_s^{*+}(2317)$ . As the structure of  $D_s^{*+}(2317)$  is not well understood, the DAs of  $D_s^{*+}(2317)$  are rather model dependent. In this paper, we employ a different sum rule for the transition form factor following Ref.[19], where the correlation function is constructed with the on-shell  $B$ -meson and the interpolated current for the charmed scalar meson. As the nonperturbative dynamics is parameterized in terms of the  $B$ -meson DAs[20, 21], the new method is usually called  $B$ -meson LCSR and it has been widely applied to the calculation of heavy-to-light matrix elements[22, 23].

In this work, we will employ the  $B$ -meson LCSR approach to evaluate the  $\bar{B}_s^0 \rightarrow D_{s0}^{*+}(2317)$  and  $B^- \rightarrow D_0^{*0}(2400)$  form factors. In our calculation  $D_{s0}^{*+}(2317)$  and  $D_0^{*0}(2400)$  are regarded as  $q\bar{q}$  mesonic states. The relevant semi-leptonic  $\bar{B}_s^0 \rightarrow D_{s0}^{*+}(2317)l\nu$  and  $B^- \rightarrow D_0^{*0}(2400)l\nu$  decay modes are also analyzed. The large number of data accumulated in the  $B$  factories and LHC-b can test whether our assumption is reasonable, and the result can help to clarify the inner structures of the new measured charmed scalar mesons.

The paper is arranged as follows: We firstly derive the LCSR for the  $\bar{B}_s^0 \rightarrow D_{s0}^{*+}(2317)$  and  $B^- \rightarrow D_0^{*0}(2400)$  form factors in the section II. The contributions from both two-particle and three-particle wave functions of  $B$  meson are computed. The numerical analysis of LCSR for the transition form factors at large recoil region is displayed in section III. The HQET is adopted to describe transitions at the small recoil region. Moreover, detailed comparisons between the form factors obtained under various approaches are also presented here. Utilizing these form factors, the

branching fractions of semileptonic decays are calculated in section IV. The last section is devoted to the conclusion.

## II. THE LIGHT-CONE SUM RULES FOR FORM FACTORS

The  $B$ -to-charmed scalar meson transition form factor induced by an axial vector current is defined by:

$$\langle D_0^*(p) | \bar{c} \gamma_\mu \gamma_5 b | \bar{B}(p+q) \rangle = -i \left\{ p_\mu f_{BD_0^*}^+(q^2) + q_\mu f_{BD_0^*}^-(q^2) \right\}, \quad (1)$$

where the notation “ $\bar{B}$ ” denotes  $\bar{B}^0, B^+$  and  $\bar{B}_s$ , and  $D_0^*$  refers to  $D_{s0}^{*+}(2317)$  and  $D_0^{*0}(2400)$ . To obtain the form factors with  $B$  meson LCSR, we consider the following correlation function with on-shell  $B$ -meson state:

$$F_\mu(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \bar{q}(x) c(x), \bar{c}(0) \gamma_\mu (1 - \gamma_5) b(0) \} | \bar{B}(P+q) \rangle, \quad (2)$$

where  $\bar{c} \gamma_\mu (1 - \gamma_5) b$  is the  $b \rightarrow c$  (electro)weak currents and  $\bar{q}c$  is the interpolating current for a charmed scalar meson.

The hadronic representation of the correlation function can be written as

$$\begin{aligned} F_\mu(p, q) &= \frac{\langle 0 | \bar{q}(0) c(0) | D_0^*(p) \rangle \langle D_0^*(p) | \bar{c}(0) \gamma_\mu \gamma_5 b(0) | \bar{B}(P+q) \rangle}{m_{D_0^*}^2 - p^2} \\ &+ \sum_h \frac{\langle 0 | \bar{q}(0) c(0) | h(p) \rangle \langle h(p) | \bar{c}(0) \gamma_\mu \gamma_5 b(0) | \bar{B}(P+q) \rangle}{s - p^2}. \end{aligned} \quad (3)$$

The decay constants  $f_{D_0^*}$  and  $\tilde{f}_{D_0^*}$  are given by

$$\langle 0 | \bar{q} \gamma_\mu c | D_0^*(p) \rangle = f_{D_0^*} p_\mu, \quad \langle 0 | \bar{q} c | D_0^*(p) \rangle = m_{D_0^*} \tilde{f}_{D_0^*}, \quad (4)$$

where  $f_{D_0^*} = (m_c - m_q) \tilde{f}_{D_0^*} / m_{D_0^*}$  and  $m_c, m_q$  are the current masses of charm quark and light quark, respectively. Inserting the definitions of the form factors and decay constants, the correlation function reads:

$$\begin{aligned} F_\mu(p, q) &= \frac{-im_{D_0^*}^2 f_{D_0^*}}{(m_c - m_q)(m_{D_0^*}^2 - p^2)} [f_{D_{s0}^*}^+(q^2) p_\mu + f_{D_0^*}^-(q^2) q_\mu] \\ &+ \int_{s_0^{D_0^*}}^\infty ds \frac{\rho_+^h(s, q^2) p_\mu + \rho_-^h(s, q^2) q_\mu}{s - p^2}, \end{aligned} \quad (5)$$

where  $s_0^{D_0^*}$  is the threshold parameter corresponding to the  $D_0^*$  channel.

On the other side, in the deep Euclidean region, the correlation function can be calculated in the perturbative theory using the operator production expansion near the light cone :

$$\begin{aligned} F_\mu(p, q) &= F_+^{\text{QCD}}(q^2, p^2) p_\mu + F_-^{\text{QCD}}(q^2, p^2) q_\mu \\ &= \int_{m_c^2}^\infty ds \frac{1}{\pi} \frac{\text{Im} F_+^{\text{QCD}}(q^2, p^2)}{s - p^2} p_\mu + \int_{m_c^2}^\infty ds \frac{1}{\pi} \frac{\text{Im} F_-^{\text{QCD}}(q^2, p^2)}{s - p^2} q_\mu. \end{aligned} \quad (6)$$

Applying the quark-hadron duality

$$\rho_i^h(s, q^2) = \frac{1}{\pi} \text{Im} F_i^{\text{QCD}}(q^2, p^2) \Theta(s - s_0^h), \quad (7)$$

with  $i = +, -$  and performing Borel transformation with respect to the variable  $p^2$ , we can derive the sum rules for the form factors as

$$f_i(q^2) = -i \frac{m_c - m_q}{\pi f_{D_0^*} m_{D_0^*}^2} \int_{m_c^2}^{s_0^h} ds \text{Im} F_i^{\text{QCD}}(q^2, s) \exp\left(\frac{m_{D_0^*}^2 - s}{M_B^2}\right). \quad (8)$$

The leading-order contribution to the OPE is illustrated in Fig. 1a. The correlation function can be calculated by contracting the charm quark fields in Eq. (2) and inserting the  $c$  quark propagator, then we arrive at:

$$F_\mu^{(B)}(p) = i \int d^4x e^{ip \cdot x} \int \frac{d^4k}{(2\pi)^4} i e^{-ik \cdot x} \langle 0 | T \{ \bar{q}(x) S_F(x, 0) \gamma_\nu (1 - \gamma_5) b(0) \} | \bar{B}(P_B) \rangle \quad (9)$$

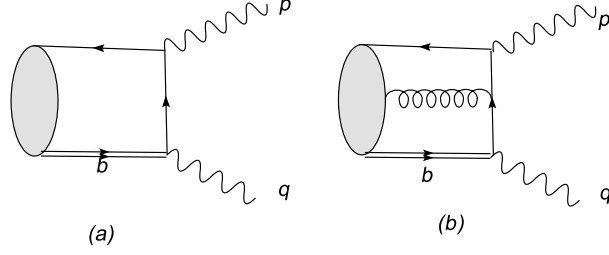


FIG. 1: Diagrams corresponding to the contributions of (a) two-particle and (b) three-particle  $B$ -meson DA's to the correlation function (2)

The full quark propagator can be written as[24],

$$S_F(x, 0)_{ij} = \delta_{ij} \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \frac{i}{\not{k} - m_c} - ig \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \int_0^1 d\alpha \left[ \frac{1}{2} \frac{\not{k} + m_c}{(m_c^2 - k^2)^2} G_{ij}^{\mu\nu}(\alpha x) \sigma_{\mu\nu} \right. \\ \left. + \frac{1}{m_c^2 - k^2} \alpha x_\mu G^{\mu\nu}(\alpha x) \gamma_\nu \right], \quad (10)$$

where the first term is the free-quark propagator and  $G_{ij}^{\mu\nu} = G_{\mu\nu}^a T_{ij}^a$  with  $\text{Tr}[T^a T^b] = \frac{1}{2} \delta^{ab}$ . Inserting this propagator to Eq.(9), we can find that the long distance contribution to the correlation function is expressed by non-local matrix elements, which defines the  $B$ -meson light-cone DA. In the leading Fock state:

$$\langle 0 | \bar{q}_{2\alpha}(x) [x, 0] h_{v\beta}(0) | \bar{B}_v \rangle \\ = -\frac{if_B m_B}{4} \int_0^\infty d\omega e^{-i\omega v \cdot x} \left[ (1 + \not{v}) \left\{ \phi_+^B(\omega) - \frac{\phi_+^B(\omega) - \phi_-^B(\omega)}{2v \cdot x} \not{x} \right\} \gamma_5 \right]_{\beta\alpha}, \quad (11)$$

where  $[x, 0]$  is the path-ordered gauge factor. The variable  $\omega > 0$  is the plus component of the spectator-quark momentum in the  $B$  meson. The three-particle DAs' contribution is shown in the diagram Fig. (1b), with the definition

$$\langle 0 | \bar{q}_{2\alpha}(x) G_{\lambda\rho}(ux) h_{v\beta}(0) | \bar{B}^0(v) \rangle = \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi e^{-i(\omega + u\xi)v \cdot x} \\ \times \left[ (1 + \not{v}) \left\{ (v_\lambda \gamma_\rho - v_\rho \gamma_\lambda) (\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)) - i\sigma_{\lambda\rho} \Psi_V(\omega, \xi) \right. \right. \\ \left. \left. - \left( \frac{x_\lambda v_\rho - x_\rho v_\lambda}{v \cdot x} \right) X_A(\omega, \xi) + \left( \frac{x_\lambda \gamma_\rho - x_\rho \gamma_\lambda}{v \cdot x} \right) Y_A(\omega, \xi) \right\} \gamma_5 \right]_{\beta\alpha}, \quad (12)$$

where the gauge link factors are omitted for brevity. The DA's  $\Psi_V, \Psi_A, X_A$  and  $Y_A$  depend on two variables  $\omega$  and  $\xi$ , corresponding to the plus components of the light-quark and gluon momenta in the  $B$  meson.

Substituting the  $B$  meson distribution function into the correlation function and employing the quark hadron duality(7), we arrive at the sum rules for transition form factors as

$$f_{BD_0^*}^+ = \frac{f_B m_B (m_c - m_s)}{f_{D_0^*} m_{D_0^*}^2} \int_0^{\sigma_0} d\sigma e^{-(s - m_{D_0^*}^2)/M^2} \left\{ m_B (\bar{\sigma} - r_c) \left( \frac{1}{\bar{\sigma}} + \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \right) \phi_+ - \frac{m_B^2 m_c (\bar{\sigma} - r_c)}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \phi_- \right. \\ \left. + \left[ -\frac{1}{\bar{\sigma}} - \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} + \frac{2m_B^3 m_c \bar{\sigma} (\bar{\sigma} - r_c)}{(\bar{\sigma}^2 m_B^2 + m_c^2 - q^2)^2} \right] \Phi_\pm \right\} + f_{BD_0^*}^{+3p}, \quad (13)$$

$$f_{BD_0^*}^- = -\frac{f_B m_B (m_c - m_s)}{f_{D_0^*} m_{D_0^*}^2} \int_0^{\sigma_0} d\sigma e^{-(s - m_{D_0^*}^2)/M^2} \left\{ m_B (\sigma + r_c) \left( \frac{1}{\bar{\sigma}} + \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \right) \phi_+ - \frac{m_B^2 m_c (\sigma + r_c)}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \phi_- \right. \\ \left. + \left[ \frac{1}{\bar{\sigma}} + \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} + \frac{2m_B^3 m_c \bar{\sigma} (\sigma + r_c)}{(\bar{\sigma}^2 m_B^2 + m_c^2 - q^2)^2} \right] \Phi_\pm \right\} + f_{BD_0^*}^{-3p}, \quad (14)$$

where the argument of the wave functions is  $m_B \sigma$ . In addition,  $\bar{\sigma} = 1 - \sigma$  and  $\sigma_0$  is the root of the equation  $\bar{\sigma} s_0 - (\sigma \bar{\sigma} + r_c^2) m_B^2 + \sigma m_B^2 q^2 = 0$ . The modified wave function  $\Phi_{\pm}(\omega) = \int_0^\omega d\tau [\phi_+(\tau) - \phi_-(\tau)]$ . The contributions from three particle  $B$  meson DAs are denoted by  $f_{BD_0^*}^{+3p}$  and  $f_{BD_0^*}^{-3p}$ , which are given in the appendix.

### III. NUMERICAL ANALYSIS OF SUM RULES FOR FORM FACTORS

Now we are going to calculate the form factors  $f_{D_0^*}(q^2)$  and  $f_{D_s^*}(q^2)$  numerically. In the following, we list the relevant input parameters for the  $D_s^+(2317)$  and  $D_0^*(2400)$ . Their mass is taken from PDG [25]:  $m_{D_{s0}^*} = 2.318\text{GeV}$  and  $m_{D_0^*} = 2.318\text{GeV}$ . The decay constant  $\tilde{f}_{D_{s0}^*} = (250 \pm 25)\text{MeV}$  [26]. For the  $D_0^*(2400)$  state, we expect  $\tilde{f}_{D_0^*}^i / \tilde{f}_{D_{s0}^*}^i = f_D^i / f_{D_s}^i$  in the SU(3) limit. We adopt the values  $f_D = (223 \pm 18)\text{MeV}$  and  $f_{D_s} = (274 \pm 20)\text{MeV}$ , we find  $\tilde{f}_{D_0^*} = (203 \pm 30)\text{MeV}$ . As for the decay constant of  $B_s$  meson, we use the results  $f_B = 130\text{MeV}$  [27] and  $f_{B_s} / f_B = 1.16 \pm 0.09$  [28] determined from QCDSR. The threshold parameter  $s_0$  can be fixed by fitting the LCSR of the charmed meson masses to the experimental data. Numerically, the threshold value in the  $X$  channel would be  $s_X^0 = (m_X + \Delta_X)^2$ , where  $\Delta_X$  is about 0.6 GeV [29–31], and we simply take it as  $(0.6 \pm 0.1)\text{GeV}$  in the error analysis. The two-particle DAs of  $B$ -meson inspired from QCD sum rule analysis reads [20]:

$$\begin{aligned}\phi_+^B(\omega) &= \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}}, \\ \phi_-^B(\omega) &= \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}},\end{aligned}\tag{15}$$

and the 3-particle DAs are given by:

$$\begin{aligned}\Psi_A(\omega, \xi) &= \Psi_V(\omega, \xi) = \frac{\lambda_E^2}{6\omega_0^4} \xi^2 e^{-(\omega+\xi)/\omega_0}, \\ X_A(\omega, \xi) &= \frac{\lambda_E^2}{6\omega_0^4} \xi(2\omega - \xi) e^{-(\omega+\xi)/\omega_0}, \\ Y_A(\omega, \xi) &= -\frac{\lambda_E^2}{24\omega_0^4} \xi(7\omega_0 - 13\omega + 3\xi) e^{-(\omega+\xi)/\omega_0}.\end{aligned}\tag{16}$$

The parameters  $\omega_0$ ,  $\lambda_H$  and  $\lambda_E$  satisfy the conditions adopted in [20]:

$$\omega_0 = \frac{2}{3}\bar{\Lambda}, \quad \lambda_E^2 = \lambda_H^2 = \frac{3}{2}\omega_0^2 = \frac{2}{3}\bar{\Lambda}^2.\tag{17}$$

Numerically we employ the values  $\omega_0^B = 0.45 \pm 0.10\text{GeV}$  and  $\omega_0^{B_s} = 0.50 \pm 0.10\text{GeV}$ , here we have taken small SU(3) breaking effect into account.

After fixing the corresponding parameters, we can proceed to compute the numerical values of the form factors. In principle, the form factors should not depend on the unphysical Borel mass  $M^2$ . However, the OPE series are truncated up to next to leading Fock state of the  $B$  meson and the QCD corrections are not considered, a manifest dependence of the form factors on the Borel parameter  $M^2$  would emerge. Therefore, we should search for the so-called ‘‘Borel window’’, where Borel mass dependence is mild, in order that the truncation is acceptable.

We firstly focus on the form factors at zero momentum transfer. To extract the form factor  $f_{D_0^*}^i(0)$ , the contribution from the higher resonances and continuum states should be less than 30 % in the total sum rules and the value of  $f_{D_0^*}^i(0)$  should not be sensitive to the Borel mass. In view of these considerations, the Borel parameter  $M^2$  should not be either too large or too small. To make sure that the contributions from the higher states are exponentially damped ( see Eq. (14)) and the global quark-hadron duality is satisfied, we need a smaller Borel mass. On the other hand, the Borel mass could not be too small for the validity of OPE near the light-cone for the correlation function, since the contributions of higher twist distribution amplitudes amount to the higher power of  $1/M^2$  to the perturbative part. In this way, we find a Borel platform  $M^2 \in [3.5, 5]\text{GeV}^2$ . The Borel mass dependence of the form factors is plotted in Fig. 2 and Fig. 3, the former includes the contribution from the three-point  $B$  meson distribution amplitudes and the higher states contribution is shown in the latter one. From these diagrams we can easily see that the higher Fock state is highly suppressed in the Borel window, and higher excited states and the continuum states contribution is within 15% for  $f_{D_0^*}^+(0)$  (30% for  $f_{D_0^*}^-(0)$ ). The numerical value for these form factors are collected in Table I, where the uncertainties are from the combination of the variation of shape parameter  $\omega_0$ , the fluctuation of threshold value, the uncertainties of quark masses and the errors of decay constants for the involved mesons. The results in the other

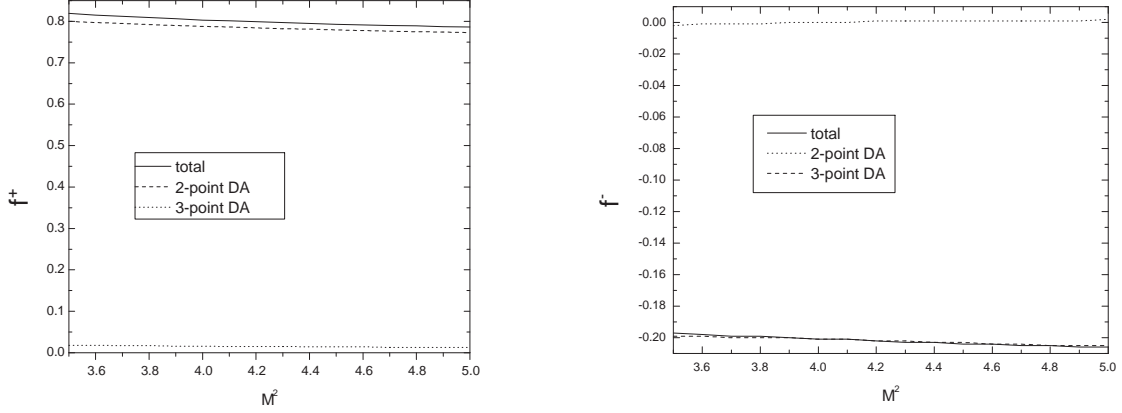


FIG. 2: The dependence form factor  $f_{D_{s0}^*}^+(0)$  and  $f_{D_{s0}^*}^-(0)$  on the Borel mass  $M^2$ , the contribution from 2-point  $B$  meson DA is denoted by the dashed line, and the dotted line represents the 3-point DA contribution. The solid line gives the total results.

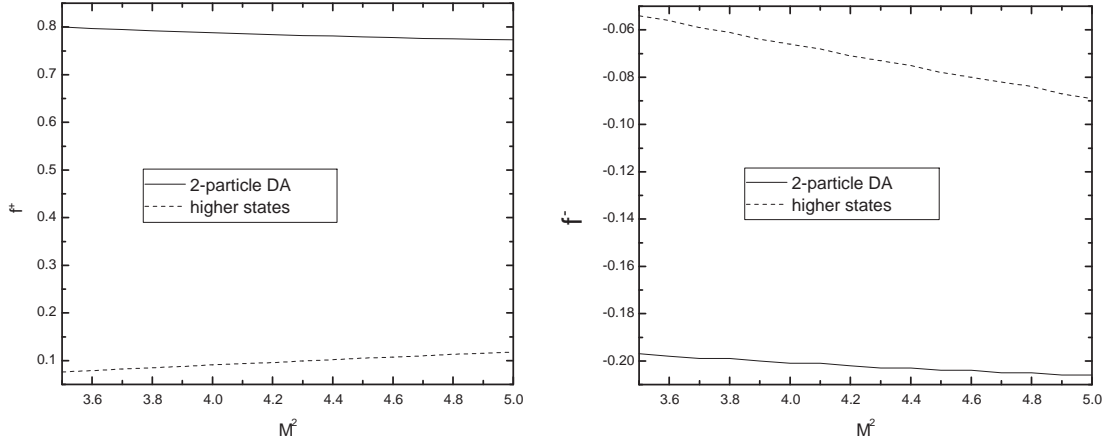


FIG. 3: The dependence form factor  $f_{D_{s0}^*}^+(0)$  and  $f_{D_{s0}^*}^-(0)$  on the Borel mass  $M^2$ , the contribution of higher excited states and the continuum states in the whole sum rules is shown by the dotted line.

studies are listed for comparison. We can see that for  $f_{D_{s0}^*}^+(0)$  our result is slightly larger than light-meson LQSR, however the results are consistent with each other within the errors. For  $f_{D_{s0}^*}^-(0)$ , the sign of our result is consistent with that obtained from the QCDSR, but it is different from that derived in the light meson LCSR. This discrepancy is expected to be smeared by power corrections.

We can also investigate the  $q^2$  dependence of the form factors  $f_{D_0^*}(q^2)$ . It is known that the OPE for the correlation function is valid only at small momentum transfer region  $0 < q^2 < (m_b - m_c)^2 - 2\Lambda_{\text{QCD}}(m_b - m_c)$ . At the large momentum transfer region, we need to parameterize them in terms of phenomenological models. To achieve this goal we firstly analyze the form factors within the HQET framework, which works well for the  $b \rightarrow c$  transition. The matrix elements responsible for  $B \rightarrow D_0^*$  transition can be parameterized as [32]

$$\langle D_0^{*+}(P) | \bar{c} \gamma_\mu \gamma_5 b | \bar{B}(P+q) \rangle = -i\sqrt{m_B m_{D_0^*}} [\eta_{D_0^*}^+(w)(v+v')_\mu + \eta_{D_0^*}^-(w)(v-v')_\mu], \quad (18)$$

TABLE I: Numbers of  $f_i^\pm(0)$  and  $\eta_i^\pm(w)$  determined from the LCSR approach, where the uncertainties from the Borel mass, threshold value, quark masses and decay constants are combined together. For comparison, the results estimated in the QCDSR are also collected here.

	this work	Light meson LQSR	QCDSR		$\eta_i^\pm(1)$	$a_i^\pm$	$b_i^\pm$
$f_{D_{s0}^*}^+(q^2)$	$0.80^{+0.24}_{-0.19}$	$0.53^{+0.12}_{-0.11}$	$0.40 \pm 0.10$ [8]	$\eta_{D_{s0}^*}^+(w)$	$0.29^{+0.08}_{-0.06}$	$-0.49^{+0.33}_{-0.54}$	$0.53^{+0.86}_{-0.56}$
$f_{D_{s0}^*}^-(q^2)$	$-0.20^{+0.08}_{-0.10}$	$0.18^{+0.06}_{-0.04}$	$-0.12 \pm 0.13$ [8]	$\eta_{D_{s0}^*}^-(w)$	$-0.86^{+0.23}_{-0.24}$	$1.59^{+0.60}_{-0.42}$	$-1.61^{+0.62}_{-0.90}$
$f_{D_0^*}^+(q^2)$	$0.94^{+0.31}_{-0.24}$	-	-	$\eta_{D_0^*}^+(w)$	$0.28^{+0.11}_{-0.07}$	$-0.34^{+0.69}_{-0.65}$	$0.32^{+1.10}_{-1.21}$
$f_{D_0^*}^-(q^2)$	$-0.27^{+0.12}_{-0.11}$	-	-	$\eta_{D_0^*}^-(w)$	$-1.01^{+0.26}_{-0.32}$	$1.86^{+1.14}_{-0.62}$	$-2.00^{+0.99}_{-1.72}$

where  $v = (P + q)/m_B$  and  $v' = P/m_{D_0^*}$  are the four-velocity vectors of  $B$  and  $D_0^*$  mesons, and  $w = v \cdot v' = (m_B^2 + m_{D_0^*}^2 - q^2)/2m_B m_{D_0^*}$ . Combining Eqs. (1) and (18), we have

$$\begin{aligned}
 f_i^+(q^2) &= \frac{1}{\sqrt{m_{B_i} m_{D_i}}} [(m_{B_i} + m_{D_i}) \eta_i^+(w) - (m_{B_i} - m_{D_i}) \eta_i^-(w)], \\
 f_i^-(q^2) &= \sqrt{\frac{m_{D_i}}{m_{B_i}}} [\eta_i^+(w) + \eta_i^-(w)],
 \end{aligned} \tag{19}$$

with  $i = 1, 2$  denotes strange and strangeless charmed scalar meson respectively. Similarly to the Isgur-Wise function  $\xi(v \cdot v')$  for the  $s$ -wave transitions, heavy quark symmetry allows to relate the form factors  $\eta_i^+(w)$  and  $\eta_i^-(w)$  to a universal function  $\tau_{1/2}(w)$  [5]

$$\eta_i^+(w) + \eta_i^-(w) = -2\tau_{1/2}(w), \quad \eta_i^+(w) - \eta_i^-(w) = 2\tau_{1/2}(w). \tag{20}$$

Different from the Isgur-Wise function  $\xi(w)$ , one can not employ the heavy quark symmetry to predict the normalization of  $\tau_{1/2}(w)$  [33].

Phenomenologically, one can parameterize the  $B \rightarrow D_0^*$  form factors in the small recoil region as

$$\eta_i^\pm(w) = \eta_i^\pm(1) + a_i^\pm(w - 1) + b_i^\pm(w - 1)^2, \tag{21}$$

The parameters  $\eta_i^\pm(1)$ ,  $a_i^\pm$  and  $b_i^\pm$  can be determined by connecting the form factors derived in the LCSR and HQET approaches in the vicinity of region with  $q^2 \sim (m_b - m_c)^2 - 2\Lambda_{\text{QCD}}(m_b - m_c)$ . In this way, we can derive the results of form factors in the whole kinematical region, in Fig. (4) we take  $f_{D_{s0}^*}^+(q^2)$  as an example. The parameters related to all the form factors are tabulated in Table I.

As discussed before, the power-suppressed form factors  $f_i^-$  in Table I suffer from sizable power corrections, which can even change the sign. Generally speaking, the corrections can be picked up by perform the heavy quark expansion of the current

$$\bar{c} \Gamma_i b = \bar{c}_{v_2} \Gamma_i b_{v_1} - \frac{1}{2m_c} \bar{c}_{v_2} \Gamma_i i \not{D}_{\perp 2} b_{v_1} + \frac{1}{2m_b} \bar{c}_{v_2} \Gamma_i i \not{D}_{\perp 1} b_{v_1} + \dots \tag{22}$$

The last two terms in the above equation might give important contribution for finite quark mass, which could help to reduce the discrepancy among different approaches. In addition, the radiative correction may also help.

#### IV. SEMILEPTONIC DECAYS

The semileptonic decays  $\bar{B}_{(s)} \rightarrow D_{0(s)}^* l \nu$  are important measurements in the  $B$  factory which can be connected with the form factors directly. The differential decay width is given by:

$$\begin{aligned}
 \frac{d\Gamma}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{768\pi^3 m_B^3} \frac{(q^2 - m_l^2)^2}{(q^2)^3} \sqrt{\lambda} \left[ (2m_l^2(\lambda + 3q^2 m_{D_0^*}^2) + q^2 \lambda) |f_i^+(q^2)|^2 \right. \\
 &\quad \left. + 6q^2 m_l^2 (m_B^2 - m_{D_0^*}^2 - q^2) f_i^+(q^2) f_i^-(q^2) + 6q^4 m_l^2 |f_i^-(q^2)|^2 \right],
 \end{aligned} \tag{23}$$

with  $\lambda = (m_B^2 - m_{D_0^*}^2 - q^2)^2 - 4q^2 m_{D_0^*}^2$ .

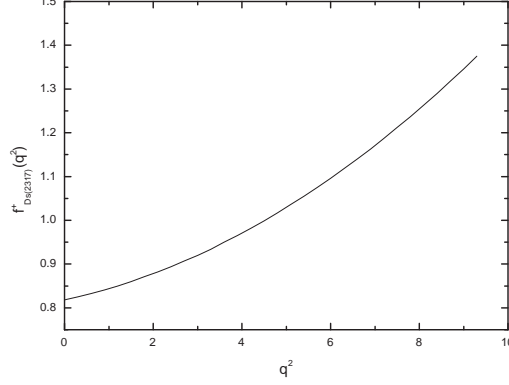


FIG. 4: The dependence of form factor  $f_{D_{s0}(2317)}^+$  on  $q^2$ ,

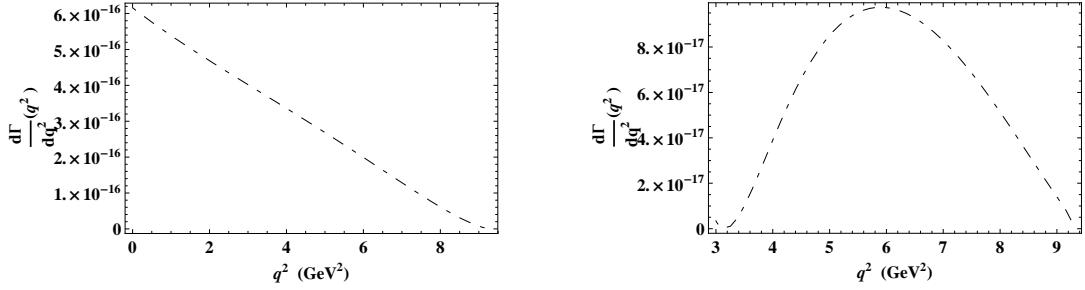


FIG. 5: The  $q^2$  dependence of differential decay width  $\frac{d}{dq^2}\Gamma(\bar{B}_s^0 \rightarrow D_{s0}^{*+}l^-\bar{\nu}_l)$  for the final states with  $l = e, \mu$  (left figure) and  $l = \tau$  (right figure).

The  $q^2$  dependence of these  $\bar{B}_s^0 \rightarrow D_{s0}^{*+}l^-\bar{\nu}_l$  partial decay rates are plotted in Fig. (5). Similar figures can also describe the  $B^- \rightarrow D_0^{*0}l^-\bar{\nu}_l$  decays. The curve of the  $\tau$  final state is different from the light quark case for its mass effect. Integrating Eq. (23), we get the branching fractions of  $\bar{B}_{(s)} \rightarrow D_{0(s)}^{*0}l\nu$  as grouped in Table II. The results from the constituent quark model, the QCD sum rules and the light quark LCSR are also listed here. Our result is slightly larger than the light quark LCSR as we have obtained large form factors. Note that the theoretical error is very large, which makes all the results are actually consistent. Besides, we can also find that the decay rates for the final state with  $\tau$  lepton are generally 3 – 4 times smaller than those for the muon case due to the suppression of phase spaces. The branching fractions for  $\bar{B}^0 \rightarrow D_0^{*0}(2400)l\bar{\nu}_l$  are also available, which is the first prediction for these decays, and we hope the future experiments can check our results.

## V. DISCUSSION AND CONCLUSION

The charmed scalar meson spectroscopy has received many research interests both experimentally and theoretically. The  $B_{(s)}$  decays provide ideal places to study their property. In this article, we employ the  $B$ -meson light-cone sum rules to compute the  $\bar{B}_s^0 \rightarrow D_{s0}^{*+}(2317)$  and  $B^- \rightarrow D_0^{*0}(2400)$  transition form factors at large recoil region, assuming  $D_{s0}^{*+}(2317)$  and  $D_0^{*0}(2400)$  being scalar quark-anti-quark states. With the help of HQET, we extrapolate the result to the whole momentum region, the  $q^2$  dependence has been plotted in Fig(4). Our results are compared with the studies using the other nonperturbative methods, such as the light-quark LCSR, the QCD sum rules and the quark models. Considering large uncertainties, our results are consistent with these studies. Meanwhile, we also found that the power corrections should be large, which even change the sign of power-suppressed form factor  $f_{D_{s0}^{*+}(2317)}^-$ .

Subsequently, we utilize the form factors obtained using  $B$ -meson LCSR to estimate the semileptonic decays  $\bar{B}_s^0 \rightarrow$



TABLE II: Branching ratios for the semileptonic decays  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)l\bar{\nu}_l$  and  $B^- \rightarrow D_0^{*0}(2400)l\bar{\nu}_l$  with the form factors estimated in  $B$ -meson LCSR, where the results calculated in the conventional light meson LCSR, the constituent quark model and QCDSR are also displayed for comparison.

$\bar{B}_s^0 \rightarrow D_{s0}^{*+}l\bar{\nu}_l$	$l = e, \mu$	$l = \tau$
this work	$(6.0 \pm 1.9) \times 10^{-3}$	$(8.2_{-2.0}^{+1.8}) \times 10^{-4}$
Light meson LCSR	$(2.3_{-1.0}^{+1.2}) \times 10^{-3}$	$(5.7_{-2.3}^{+2.8}) \times 10^{-4}$
QCDSR[8]	$\sim 10^{-3}$	$\sim 10^{-4}$
Constituent Quark Model[6]	$(4.90 - 5.71) \times 10^{-3}$	
QCDSR in HQET[7]	$(0.9 - 2.0) \times 10^{-3}$	
$B^- \rightarrow D_0^{*0}l\bar{\nu}_l$	$l = e, \mu$	$l = \tau$
this work	$(8.7_{-2.8}^{+5.1}) \times 10^{-3}$	$(1.1_{-0.3}^{+0.6}) \times 10^{-3}$

$D_s^{*+}(2317)l\bar{\nu}_l$  and  $B^- \rightarrow D_0^{*0}(2400)l\bar{\nu}_l$ . It has been shown in this work that the branching fraction of the semileptonic  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)l\bar{\nu}_l$  decay is around  $6 \times 10^{-3}$  for light leptons and  $0.8 \times 10^{-3}$  for tau final state. The difference is due to the phase space suppression. The predicted values can confront with the future LHCb measurements. The predicted branching ration of  $B^- \rightarrow D_0^{*0}(2400)l\bar{\nu}_l$  is slightly larger than  $\bar{B}_s^0 \rightarrow D_s^{*+}(2317)l\bar{\nu}_l$ , and this observation can be tested at both LHCb and super  $B$  factories.

### Acknowledgement

This work is partly supported by National Natural Science Foundation of China under the Grant No. 11005100, 10735080 and 11075168. This research is also supported in part by the Project of Knowledge Innovation Program(PKIP) of Chinese Academy of Sciences, Grant No. KJCX2.YW.W10.

### Appendix

In the following we show the form factors from the 3-point B meson DA.

$$\begin{aligned}
f_{\pm}^{3p} = & \frac{f_B(m_c - m_q)}{2f_{D_0^*}m_{D_0^*}} \left\{ \int_0^{\eta_0 m_B} d\omega \int_{\eta_0 m_B - \omega}^{\infty} \frac{d\xi}{\xi} e^{-(s_0 - m_{D_0^{*2}})/M_B^2} \right. \\
& \times f(\eta_0)(m_B A_1^{\pm} + m_c A_2^{\pm} + A_3^{\pm} - A_4^{\pm} + A_5^{\pm} - \frac{A_6^{\pm} + A_7^{\pm} + m_B A_8^{\pm}}{M_B^2}) \\
& + \int_0^{\eta_0} \frac{d\eta}{\bar{\eta}^2} \int_0^{\eta m_B} d\omega \int_{\eta m_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{1}{M_B^2} e^{-(s - m_{D_0^{*2}})/M_B^2} \\
& \times (m_B B_1^{\pm} + m_c B_2^{\pm} + B_3^{\pm} - B_4^{\pm} + B_5^{\pm} - \frac{B_6^{\pm} + B_7^{\pm} + m_B B_8^{\pm}}{2M_B^2}) \\
& \left. - \frac{f(\eta_0)e^{-(s_0 - m_{D_0^{*2}})/M_B^2}}{2m_B^3} \int_0^{\eta_0 m_B} d\omega \int_{\eta_0 m_B - \omega}^{\infty} \frac{d\xi}{\xi} (C_4^{\pm} + C_6^{\pm} + C_7^{\pm} + C_8^{\pm}) \right\}, \tag{24}
\end{aligned}$$



where the functions  $A_i^\pm (i = 1, 2, \dots, 8)$ ,  $A_i^\pm (i = 1, 2, \dots, 8)$  and  $A_i^\pm (i = 1, 2, \dots, 8)$  entering the integration are given below:

$$\begin{aligned}
A_1^+ &= \left[ \frac{2\alpha_0(2 - 3\eta_0 + (s_0 - q^2)/m_B^2)}{m_B\bar{\eta}_0^2} - \frac{1 + 2(s_0 - q^2)/m_B^2 + 3(r_c - \eta_0)}{m_B\bar{\eta}_0^2} \right] (\psi_A - \psi_V) \\
A_2^+ &= \frac{6\alpha_0\bar{\eta}_0 - 6r_c}{m_B^2\bar{\eta}_0^2} \psi_V \\
A_3^+ &= \frac{-2\alpha_0}{m_B\bar{\eta}_0^2} \bar{X}_A \\
A_4^+ &= \frac{\alpha_0(-m_B^2\bar{\eta}_0^2 + m_c^2 + q^2)}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_5^+ &= \frac{\bar{\eta}_0 + 2r_c}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_6^+ &= \frac{2[m_B^2(\bar{\eta}_0 - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}_0}](\bar{\eta}_0 + r_c)}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_7^+ &= \frac{-24\alpha_0 m_c^2}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_8^+ &= \frac{(-24m_c)[m_B^2(\bar{\eta}_0 - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}_0}](\bar{\eta}_0 + r_c)}{m_B\bar{\eta}_0^3} \bar{Y}_A \\
A_1^- &= \left[ \frac{2\alpha_0(1 - 3\eta_0 + (s_0 - q^2)/m_B^2)}{m_B\bar{\eta}_0^2} - \frac{2 + 2(s_0 - q^2)/m_B^2 + 3(r_c - \eta_0)}{m_B\bar{\eta}_0^2} \right] (\psi_A - \psi_V) \\
A_2^- &= \frac{-6\alpha_0\eta_0 - 6r_c}{m_B^2\bar{\eta}_0^2} \psi_V \\
A_3^- &= \frac{2\alpha_0(1 + \eta_0)}{m_B\bar{\eta}_0^2} \bar{X}_A \\
A_4^- &= \frac{\alpha_0(m_B^2\eta_0\bar{\eta}_0 + \frac{1+\eta_0}{\bar{\eta}_0}m_c^2 - \frac{\eta_0}{\bar{\eta}_0}q^2)}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_5^- &= \frac{1 + \eta_0 - 2r_c}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_6^- &= \frac{2[m_B^2(\eta_0 - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}_0}](-\eta_0 + r_c)}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_7^- &= \frac{-24\alpha_0 m_c^2}{m_B\bar{\eta}_0^3} \bar{X}_A \\
A_8^- &= \frac{(-24m_c)[m_B^2(\eta_0 - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}_0}](-\eta_0 + r_c)}{m_B\bar{\eta}_0^3} \bar{Y}_A
\end{aligned} \tag{25}$$

$$\begin{aligned}
B_1^+ &= 2\alpha(2 - 3\eta + (s - q^2)/m_B^2) - (1 + 2(s - q^2)/m_B^2 + 3(r_c - \eta))(\psi_A - \psi_V) \\
B_2^+ &= (6\alpha\bar{\eta} - 6r_c)\psi_V \\
B_3^+ &= (-2\alpha m_B)\bar{X}_A \\
B_4^+ &= 2\alpha m_B(-m_B^2\bar{\eta}^2 + m_c^2 + q^2)\bar{X}_A \\
B_5^+ &= (\bar{\eta} + 2r_c)m_B\bar{X}_A \\
B_6^+ &= 2[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}}](\bar{\eta} + r_c)\bar{X}_A \\
B_7^+ &= (-24\alpha m_B m_c^2)\bar{X}_A \\
B_8^+ &= (-24m_c)[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}}](\bar{\eta} + r_c)\bar{Y}_A
\end{aligned} \tag{26}$$

$$\begin{aligned}
B_1^- &= 2\alpha(1 - 3\eta + (s - q^2)/m_B^2) - (2 + 2(s - q^2)/m_B^2 + 3(r_c - \eta))(\psi_A - \psi_V) \\
B_2^- &= (-6\alpha\eta - 6r_c)\psi_V \\
B_3^- &= \frac{2\alpha(1 + \eta)}{m_B\bar{\eta}^2}\bar{X}_A \\
B_4^- &= 2\alpha m_B(m_B^2\eta\bar{\eta} + \frac{1 + \bar{\eta}}{\bar{\eta}}m_c^2 - \frac{\eta}{\bar{\eta}}q^2)\bar{X}_A \\
B_5^- &= (1 + \eta - 2r_c)m_B\bar{X}_A \\
B_6^- &= 2[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}}](-\eta + r_c)\bar{X}_A \\
B_7^- &= (-24\alpha m_B m_c^2)\bar{X}_A \\
B_8^- &= (-24m_c)[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}}](\bar{\eta} + r_c)\bar{Y}_A
\end{aligned} \tag{27}$$

$$\begin{aligned}
C_4^+ &= \frac{d}{d\eta}[2\alpha(-m_B^2\bar{\eta}^2 + m_c^2 + q^2)\frac{f(\eta)\bar{X}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_6^+ &= \frac{d}{d\eta}[2[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}}](\bar{\eta} + r_c)\frac{f(\eta)\bar{X}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_7^+ &= \frac{d}{d\eta}[(-24\alpha m_c^2)\frac{f(\eta)\bar{Y}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_8^+ &= \frac{d}{d\eta}[(-24m_c)[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{-\eta}](\bar{\eta} + r_c)\frac{f(\eta)\bar{Y}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_4^- &= \frac{d}{d\eta}[2\alpha(m_B^2\eta\bar{\eta} + \frac{1 + \bar{\eta}}{\bar{\eta}}m_c^2 - \frac{\eta}{\bar{\eta}}q^2)\frac{f(\eta)\bar{X}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_6^- &= \frac{d}{d\eta}[2[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{\bar{\eta}}](-\eta + r_c)\frac{f(\eta)\bar{X}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_7^- &= \frac{d}{d\eta}[(-24\alpha m_c^2)\frac{f(\eta)\bar{Y}_A}{\bar{\eta}^3}]_{\eta=\eta_0} \\
C_8^- &= \frac{d}{d\eta}[(-24m_c)[m_B^2(\bar{\eta} - 2r_c) + \frac{m_c^2 - q^2}{-\eta}](\bar{\eta} + r_c)\frac{f(\eta)\bar{Y}_A}{\bar{\eta}^3}]_{\eta=\eta_0}
\end{aligned} \tag{28}$$

Where the notations  $\eta = \omega + \xi\alpha$ ,  $f(\eta) = \left(1 + \frac{m^2 - q^2}{\bar{\eta}^2 m_B^2}\right)^{-1}$ ,  $\bar{X}_A(\omega, \xi) = \int_0^\omega d\tau X_A(\tau, \xi)$ ,  $\bar{Y}_A(\eta, \xi) = \int_0^\omega d\tau Y_A(\tau, \xi)$ , and  $\eta_0$  satisfies the equation  $\bar{\eta}s_0 - (\eta\bar{\eta} + r_c^2)m_B^2 + \eta m_B^2 = 0$ .

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